

CHAPTER 1

TIMSS 2015 Mathematics Framework

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All children can benefit from studying and developing strong skills in mathematics. Primarily, learning mathematics improves problem-solving skills, and working through problems can teach persistence and perseverance. Mathematics is essential in daily life for such activities as counting, cooking, managing money, and building things. Beyond that, many career fields require a strong mathematical foundation, such as engineering, architecture, accounting, banking, business, medicine, ecology, and aerospace. Mathematics is vital to economics and finance, as well as to the computing technology and software development underlying our technologically advanced and information-based world.

This chapter presents the assessment frameworks for the three TIMSS 2015 mathematics assessments:

- TIMSS Mathematics—Fourth Grade;
- TIMSS Numeracy, a less difficult version of TIMSS Mathematics—Fourth Grade that is newly developed for TIMSS 2015; and
- TIMSS Mathematics—Eighth Grade.

As described in the Introduction, the TIMSS 2015 Mathematics Frameworks for the fourth and eighth grades build on TIMSS' 20-year history of assessments every four years since 1995, with this being the sixth assessment in the series. In contrast, TIMSS Numeracy is newly developed for 2015 as an alternative or supplement to TIMSS Mathematics—Fourth Grade for countries where many children are still developing fundamental mathematics skills.

In general, the fourth and eighth grade frameworks are similar to those used in TIMSS 2011. However, there have been minor updates to particular

topics to better reflect the curricula, standards, and frameworks of the participating countries as reported in the *TIMSS 2011 Encyclopedia* (Mullis et al., 2012). Also, attention was paid to current international research and initiatives concerning mathematics and education, such as the *Common Core State Standards for Mathematics* (National Governors Association, 2010) developed in the United States, the *Mathematics (Primary and Lower Secondary) Syllabi* (Singapore Ministry of Education, 2006) used in Singapore, and the *Mathematics Curriculum Guide (Primary 1–Secondary 3)* (Education Bureau, Hong Kong SAR, 2002) used in Hong Kong.

Each of the three assessment frameworks for TIMSS 2015 is organized around two dimensions:

- Content dimension, specifying the subject matter to be assessed; and
- Cognitive dimension, specifying the thinking processes to be assessed.

Exhibit 2 shows the target percentage of testing time devoted to each content and cognitive domain for the TIMSS 2015 fourth and eighth grade assessments.

Exhibit 2: Target Percentages of the TIMSS 2015 Mathematics Assessment Devoted to Content and Cognitive Domains at the Fourth and Eighth Grades¹

Fourth Grade		
Content Domains	Percentages	
Number	50%	
Geometric Shapes and Measures	35%	
Data Display	15%	
Eighth Grade		
Content Domains	Percentages	
Number	30%	
Algebra	30%	
Geometry	20%	
Data and Chance	20%	
Cognitive Domains		
	Fourth Grade	Eighth Grade
Knowing	40%	35%
Applying	40%	40%
Reasoning	20%	25%

¹ The target percentages for TIMSS Numeracy are presented on page 19.

The content domains differ for the fourth and eighth grades, reflecting the mathematics widely taught at each grade. There is more emphasis on number at the fourth grade than at the eighth grade. At the eighth grade, two of the four content domains are algebra and geometry. Because these generally are not taught as separable areas in primary school, the introductory or pre-algebra topics assessed at the fourth grade are included as part of number, while the geometric domain focuses on geometric shapes and measures. The fourth grade data domain focuses on reading and displaying data whereas at the eighth grade it includes more emphasis on interpretation of data and the fundamentals of probability (called “chance”).

It is important to highlight that TIMSS assesses a range of problem-solving situations within mathematics, with about two-thirds of the items requiring students to use applying and reasoning skills. The cognitive domains are the same for both grades, but with a shift in emphasis. Compared to the fourth grade, the eighth grade has less emphasis on the knowing domain and greater emphasis on the reasoning domain.

Following this brief introduction, the chapter begins with the fourth grade content domains, identifying the three main content domains and the assessment topics within each domain. As an option or supplement to TIMSS Mathematics—Fourth Grade, the TIMSS Numeracy Framework is presented in its entirety following the description of the fourth grade content domains, because it has been adapted from the fourth grade content domains but with less difficult topics. For ease of reference, the assessment emphasis for the cognitive domains also is included for TIMSS Numeracy. Next, Chapter 1 continues with the description of the TIMSS Mathematics—Eighth Grade content domains and, then, the descriptions of the cognitive domains for both the fourth and eighth grades.

Mathematics Content Domains—Fourth Grade

Exhibit 3 shows the TIMSS Mathematics—Fourth Grade content domains and the target percentages of testing time devoted to each. Each content domain consists of topic areas, and each topic area in turn includes several topics. Across the fourth grade mathematics assessment, each topic receives approximately equal weight in terms of time allocated to assessing the topic.

Exhibit 3: Target Percentages of the TIMSS 2015 Mathematics Assessment Devoted to Content Domains at the Fourth Grade

Fourth Grade Content Domains	Percentages
Number	50%
Geometric Shapes and Measures	35%
Data Display	15%

Number

The number content domain consists of understandings and skills related to three topic areas. The number domain at the fourth grade is the only domain where items within the domain are not distributed equally across the TIMSS 2015 content topic areas. The fifty percent of the assessment devoted to Number is apportioned as follows:

- Whole numbers (25%);
- Fractions and decimals (15%); and
- Expressions, simple equations, and relationships (10%).

Because whole numbers provide the easiest introduction to operations with numbers, working with whole numbers provides the foundation of mathematics in primary school. As such, whole numbers are the predominant component of the number domain and students should be able to compute with whole numbers of reasonable size as well as use computation to solve problems. However, because objects and quantities often do not come in whole numbers, it is also important for students to understand fractions as the basis for many calculations. Students should be able to compare familiar fractions and decimals. In addition, at the fourth grade, pre-algebraic concepts also are part of the TIMSS assessment, including understanding the concept of variable (unknowns) in simple equations, and initial understandings of relationships between quantities.

Number: Whole Numbers

1. Demonstrate knowledge of place value, including recognizing and writing numbers in expanded form; and represent whole numbers using words, diagrams, or symbols.
2. Compare, order, and round whole numbers.
3. Compute (+, −, ×, ÷) with whole numbers.

4. Solve problems set in contexts, including those involving measurements, money, and simple proportions.
5. Identify odd and even numbers; identify multiples and factors of numbers.

Number: Fractions and Decimals

1. Recognize fractions as parts of wholes, parts of a collection, or locations on number lines, and represent fractions using words, numbers, or models.
2. Identify equivalent simple fractions; compare and order simple fractions; add and subtract simple fractions, including those set in problem situations.
3. Demonstrate knowledge of decimal place value including representing decimals using words, numbers, or models; compare, order, and round decimals; add and subtract decimals, including those set in problem situations.

Note: Fourth grade fractions items will involve denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100. Fourth grade decimals items will involve decimals up to one and/or two places.

Number: Expressions, Simple Equations, and Relationships

1. Find the missing number or operation in a number sentence (e.g., $17 + w = 29$).
2. Identify or write expressions or number sentences to represent problem situations involving unknowns.
3. Identify and use relationships in a well-defined pattern (e.g., describe the relationship between adjacent terms and generate pairs of whole numbers given a rule).

Geometric Shapes and Measures

We are surrounded by objects of different shapes and sizes, and geometry helps us visualize and understand the relationships between shapes and sizes. This topic area deals with understanding measurements, the coordinate plane, lines, and angles. It also covers surfaces and solids.

The two topic areas in geometric shapes and measures are as follows:

- Points, lines, and angles; and
- Two- and three-dimensional shapes.

At the fourth grade, students should be able to identify the properties and characteristics of lines, angles, and a variety of geometric figures, including two- and three-dimensional shapes. Spatial sense is integral to the study of geometry, and students will be asked to describe and draw a variety of geometric figures. They also should be able to analyze geometric relationships and use these relationships to solve problems. Students should be able to use instruments and tools to measure physical attributes such as length, angle, area, and volume; and to use simple formulas to calculate areas and perimeters of squares and rectangles.

Geometric Shapes and Measures: Points, Lines, and Angles

1. Measure and estimate lengths.
2. Identify and draw parallel and perpendicular lines.
3. Identify, compare, and draw different types of angles (e.g., a right angle, and angles larger or smaller than a right angle).
4. Use informal coordinate systems to locate points in a plane.

Geometric Shapes and Measures: Two- and Three-dimensional Shapes

1. Use elementary properties to describe and compare common two- and three-dimensional geometric shapes, including line and rotational symmetry.
2. Relate three-dimensional shapes with their two-dimensional representations.
3. Calculate perimeters of polygons; calculate areas of squares and rectangles; and estimate areas and volumes of geometric figures by covering with a given shape or by filling with cubes.

Note: Fourth grade geometric shapes items will involve circles, triangles, quadrilaterals, and other polygons, as well as cubes, rectangular solids, cones, cylinders, and spheres.

Data Display

The explosion of data in today's information society has resulted in a daily bombardment of visual displays of quantitative information. Often the Internet, newspapers, magazines, textbooks, reference books, and articles have data represented in charts, tables, and graphs. Students need to understand that graphs and charts help organize information or categories and provide a way to compare data.

The data display content domain consists of one topic area:

- Reading, interpreting, and representing.

At the fourth grade, students should be able to read and recognize various forms of data displays. Given a simple problem situation and the data that has been collected, students should be able to organize and represent the data in graphs and charts that address the questions that prompted the data collection. Students should be able to compare characteristics of data and to draw conclusions based on data displays.

Data Display: Reading, Interpreting, and Representing

1. Read, compare, and represent data from tables, pictographs, bar graphs, line graphs and pie charts.
2. Use information from data displays to answer questions that go beyond directly reading the data displayed (e.g., solve problems and perform computations using the data, combine data from two or more sources, make inferences, and draw conclusions based on the data).

TIMSS Numeracy

To provide a broad range of countries with avenues to measure numeracy learning outcomes, TIMSS Numeracy is being introduced in 2015. It is important to match the TIMSS Mathematics assessment to a country's educational development, and TIMSS Numeracy assesses the fundamental mathematical knowledge, procedures, and problem-solving strategies that are prerequisites for success on TIMSS Mathematics—Fourth Grade.

TIMSS Numeracy asks students to answer questions and work problems similar to TIMSS Mathematics—Fourth Grade, except with easier numbers and more straightforward procedures. TIMSS Numeracy includes many of the same domains and topics as TIMSS Mathematics—Fourth Grade, but it is designed to assess mathematics at the end of the primary school cycle (4th, 5th, or 6th grades) for countries where most children are still developing fundamental mathematics skills.

Exhibit 4 shows the target percentages of testing time devoted to each of the TIMSS Numeracy content and cognitive domains. The TIMSS Numeracy content domains are described following Exhibit 4. The descriptions of the cognitive domains are the same as for the fourth and eighth grade and can be found in the last section of this chapter.

Exhibit 4: Target Percentages of the TIMSS Numeracy 2015 Mathematics Assessment Devoted to Content and Cognitive Domains

Numeracy Content Domains	Percentages
Whole Numbers	50%
Fractions and Decimals	15%
Shapes and Measures	35%

Numeracy Cognitive Domains	Percentages
Knowing	50%
Applying	35%
Reasoning	15%

Whole Numbers

1. Demonstrate knowledge of numbers (through thousands) including representing numbers, understanding place value, and ordering numbers.
2. Add and subtract whole numbers and demonstrate knowledge of these operations in simple problem settings.
3. Multiply and divide whole numbers by one-digit numbers and demonstrate knowledge of these operations in simple problem settings.
4. Read data from tables, bar graphs, and pictographs; and use the data to solve simple problems.
5. Solve problems with whole numbers including those involving more than one operation, patterns, and simple number sentences.

Fractions and Decimals

1. Recognize, compare, add, and subtract simple fractions (halves, thirds, fourths, fifths, sixths, eighths, and tenths).
2. Demonstrate knowledge of decimals including place value and ordering; and add and subtract one-place decimals.

Shapes and Measures

1. Identify and compare common geometric figures (lines, angles, and basic two- and three-dimensional shapes).
2. Compare, measure, and estimate lengths, areas, and volumes.
3. Solve problems involving measurements, including time and money.

Mathematics Content Domains—Eighth Grade

Exhibit 5 shows the TIMSS Mathematics—Eighth Grade content domains and the target percentages of testing time devoted to each. Each content domain consists of topic areas, and each topic area in turn includes several topics. Across the eighth grade mathematics assessment, each topic receives approximately equal weight in terms of time allocated to assessing the topic.

Exhibit 5: Target Percentages of the TIMSS 2015 Mathematics Assessment Devoted to Content Domains at the Eighth Grade

Eighth Grade Content Domains	Percentages
Number	30%
Algebra	30%
Geometry	20%
Data and Chance	20%

Number

At the eighth grade, the number domain consists of three topic areas:

- Whole numbers;
- Fractions, decimals, and integers; and
- Ratio, proportion, and percent.

Building on the number content domain at the fourth grade, eighth grade students should have developed proficiency with more complex whole number concepts and procedures as well as extended their mathematical understanding of rational numbers (fractions, decimals, and integers). Fractions and decimals are an important part of daily life and being able to compute with them requires an understanding of the quantities the symbols represent. Students should understand that fractions and decimals are single entities like whole numbers, and hold unique places on the number line. Students also should understand and be able to compute with integers, through movement on the number line or various models (e.g., thermometers, losses and gains). Rational numbers can be expressed in various forms, including ratios, proportions, and percents. A single rational number can be represented with many different written symbols, and students need to be able to recognize the distinctions among interpretations of rational numbers, construct relations among them, and reason with them.

Number: Whole Numbers

1. Demonstrate understanding of whole numbers and operations (e.g., the four arithmetic operations; place value; and the commutative, associative, and distributive properties).
2. Compute with whole numbers in problem situations.
3. Find and use multiples or factors of numbers, identify prime numbers, and evaluate powers of numbers and square roots of perfect squares up to 144.

Number: Fractions, Decimals, and Integers

1. Identify, compare, or order rational numbers (fractions, decimals, and integers) using various models and representations (e.g., number line); and know that there are numbers that are not rational.
2. Compute with rational numbers (fractions, decimals, and integers) including those set in problem situations.

Number: Ratio, Proportion, and Percent

1. Identify and find equivalent ratios; and model a given situation by using a ratio and divide a quantity in a given ratio.
2. Convert among percents, proportions, and fractions.
3. Solve problems involving percents or proportions.

Algebra

The topic areas in algebra are as follows:

- Expressions and operations;
- Equations and inequalities; and
- Relationships and functions.

Algebra is pervasive in the world around us, enabling patterns to be expressed as formulas so that computations do not need to be done again and again, and so that generalizations can be made about relationships. Students should be able to solve real-world problems using algebraic models and explain relationships involving algebraic concepts. They need to go beyond memorization to understand that when there is a formula about two

quantities, if they know one, they can find the other quantity. This conceptual understanding can extend to linear equations for calculations about things that expand at constant rates (e.g., slope) and quadratic expressions to study motion (e.g., the paths of traveling objects such as rockets, comets, and baseballs). Functions are studied to find out what will happen to a variable over time, including when the variable will reach its highest or lowest value.

Algebra: Expressions and Operations

1. Find the value of an expression given values of the variables.
2. Simplify algebraic expressions involving sums, products, and powers of expressions; and compare expressions to determine if they are equivalent.
3. Use expressions to represent problem situations.

Algebra: Equations and Inequalities

1. Write equations or inequalities to represent situations.
2. Solve linear equations, linear inequalities, and simultaneous linear equations in two variables.

Algebra: Relationships and Functions

1. Generalize pattern relationships in a sequence, or between adjacent terms, or between the sequence number of the term and the term, using numbers, words, or algebraic expressions.
2. Interpret, relate, and generate representations of functions in tables, graphs, or words.
3. Identify functions as linear or non-linear; contrast properties of functions from tables, graphs, or equations; and interpret the meanings of slope and y-intercept in linear functions.

Geometry

Extending the understandings of shapes and measures assessed at the fourth grade, students should be able to analyze the properties and characteristics of a variety of two- and three-dimensional figures and be competent in geometric measurement (perimeters, areas, and volumes). They should be able to solve problems and provide explanations based on geometric relationships.

The three topic areas in geometry are as follows:

- Geometric shapes;
- Geometric measurement; and
- Location and movement.

Geometry: Geometric Shapes

1. Identify different types of angles and use the relationships between angles on lines and in geometric figures.
2. Identify geometric properties of two- and three-dimensional shapes, including line and rotational symmetry.
3. Identify congruent triangles and quadrilaterals and their corresponding measures; and identify similar triangles and use their properties.
4. Relate three-dimensional shapes with their two-dimensional representations (e.g., nets, two-dimensional views of three-dimensional objects).
5. Use geometric properties, including the Pythagorean Theorem, to solve problems.

Note: Eighth grade geometric shapes items will include circles; scalene, isosceles, equilateral, and right-angled triangles; trapezoids, parallelograms, rectangles, rhombuses, and square quadrilaterals; as well as other polygons including pentagons, hexagons, octagons, and decagons. It also includes three-dimensional objects—prisms, pyramids, cones, cylinders, and spheres.

Geometry: Geometric Measurement

1. Draw and estimate the size of given angles, line segments, and perimeters; and estimate areas and volumes.
2. Select and use appropriate measurement formulas for perimeters, circumferences, areas, surface areas, and volumes; and find measures of compound areas.

Geometry: Location and Movement

1. Locate points and solve problems involving points in the Cartesian plane.
2. Recognize and use geometric transformations (translation, reflection, and rotation) of two-dimensional shapes.

Data and Chance

Increasingly, the more traditional forms of data display (e.g., bar graphs, line graphs, pie graphs, pictographs) are becoming more complicated and are being

supplanted by an array of new graphic forms. By the eighth grade, students should to be able to read and extract the important meaning from a variety of visual displays. It is also important for eighth grade students to be familiar with the statistics underlying data distributions (e.g., mean, median, mode, and spread) and how these relate to the shape of data graphs. In order to avoid being misled by distorted representations of data, students also should understand how the creators of charts and graphics can misrepresent the truth. Finally, students should have an initial grasp of some concepts related to probability.

The data and chance content domain contains three topic areas:

- Characteristics of data sets;
- Data interpretation; and
- Chance.

Data and Chance: Characteristics of Data Sets

1. Identify and compare characteristics of data sets including mean, median, mode, range, and shape of distributions (in general terms).
2. Calculate, use, or interpret mean, median, mode, or range to solve problems.

Data and Chance: Data Interpretation

1. Read data from a variety of visual data displays.
2. Use and interpret data sets to solve problems (e.g., make inferences, draw conclusions, and estimate values between and beyond given data points).
3. Identify and describe approaches to organizing and displaying data that could lead to misinterpretation (e.g., inappropriate grouping, and misleading or distorted scales).

Data and Chance: Chance

1. Judge chances of outcomes as certain, more likely, equally likely, less likely, or impossible in general terms.
2. Use data, including experimental data, to estimate the chances of future outcomes.
3. Given a process designed to be random, determine the chances of possible outcomes.

Calculator Use at the Eighth Grade

Although technology in the form of calculators and computers can help students learn mathematics, it should not be used to replace basic understanding and competencies. Like any teaching tool, calculators need to be used appropriately, and policies for their use differ across the TIMSS countries. Also, the availability of calculators varies widely. It would not be equitable to require calculator use when students in some countries may never have used them. Similarly, however, it is not equitable to deprive students of the use of a familiar tool.

In order to give students the best opportunity to operate in settings that mirror their classroom experience, TIMSS has permitted calculator use at the eighth grade since 2003. Thus, if eighth grade students are accustomed to having calculators for their classroom activities, then the country should encourage students to use them during the assessment. On the other hand, if students are not accustomed to having calculators or are not permitted to use them in their daily mathematics lessons, then the country need not permit their use. In developing the new assessment materials, every effort will be made to ensure that the test questions do not advantage or disadvantage students either way, with or without calculators.

Mathematics Cognitive Domains—Fourth and Eighth Grades

In order to respond correctly to TIMSS test items, students need to be familiar with the mathematics content being assessed, but they also need to draw on a range of cognitive skills. Describing these skills plays a crucial role in the development of an assessment like TIMSS 2015, because they are vital in ensuring that the survey covers the appropriate range of cognitive skills across the content domains already outlined.

The first domain, *knowing*, covers the facts, concepts, and procedures students need to know, while the second, *applying*, focuses on the ability of students to apply knowledge and conceptual understanding to solve problems or answer questions. The third domain, *reasoning*, goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multi-step problems.

These three cognitive domains are used for both grades, but the balance of testing time differs, reflecting the difference in age and experience of students in the two grades. For the fourth and eighth grades, each content domain will include items developed to address each of the three cognitive domains. For example, the number domain will include knowing, applying, and reasoning items as will the other content domains.

Exhibit 6 shows the target percentages of testing time devoted to each cognitive domain for the fourth and eighth grade assessments.

Exhibit 6: Target Percentages of the TIMSS 2015 Mathematics Assessment Devoted to Cognitive Domains at the Fourth and Eighth Grades

Cognitive Domains	Percentages	
	Fourth Grade	Eighth Grade
Knowing	40%	35%
Applying	40%	40%
Reasoning	20%	25%

Knowing

Facility in applying mathematics, or reasoning about mathematical situations, depends on familiarity with mathematical concepts and fluency in mathematical skills. The more relevant knowledge a student is able to recall and the wider the range of concepts he or she understands, the greater the potential for engaging in a wide range of problem-solving situations.

Without access to a knowledge base that enables easy recall of the language and basic facts and conventions of number, symbolic representation, and spatial relations, students would find purposeful mathematical thinking impossible. Facts encompass the knowledge that provides the basic language of mathematics, as well as the essential mathematical concepts and properties that form the foundation for mathematical thought.

Recall	Recall definitions, terminology, number properties, units of measurement, geometric properties, and notation (e.g., $a \times b = ab$, $a + a + a = 3a$).
Recognize	Recognize numbers, expressions, quantities, and shapes. Recognize entities that are mathematically equivalent (e.g., equivalent familiar fractions, decimals, and percents; different orientations of simple geometric figures).
Classify/Order	Classify numbers, expressions, quantities, and shapes by common properties.
Compute	Carry out algorithmic procedures for $+$, $-$, $\times$, $\div$, or a combination of these with whole numbers, fractions, decimals, and integers. Carry out straightforward algebraic procedures.
Retrieve	Retrieve information from graphs, tables, texts, or other sources.
Measure	Use measuring instruments; and choose appropriate units of measurement.

Procedures form a bridge between more basic knowledge and the use of mathematics for solving problems, especially those encountered by many people in their daily lives. In essence, a fluent use of procedures entails recall of sets of actions and how to carry them out. Students need to be efficient and accurate in using a variety of computational procedures and tools. They need to see that particular procedures can be used to solve entire classes of problems, not just individual problems.

Applying

The applying domain involves the application of mathematics in a range of contexts. In this domain, the facts, concepts, and procedures as well as the problems should be familiar to the student. In some items aligned with this domain, students need to apply mathematical knowledge of facts, skills, and procedures or understanding of mathematical concepts to create representations. Representation of ideas forms the core of mathematical thinking and communication, and the ability to create equivalent representations is fundamental to success in the subject.

Problem solving is central to the applying domain, with an emphasis on more familiar and routine tasks. Problems may be set in real-life situations, or may be concerned with purely mathematical questions involving, for example, numeric or algebraic expressions, functions, equations, geometric figures, or statistical data sets.

Determine	Determine efficient/appropriate operations, strategies, and tools for solving problems for which there are commonly used methods of solution.
Represent/Model	Display data in tables or graphs; create equations, inequalities, geometric figures, or diagrams that model problem situations; and generate equivalent representations for a given mathematical entity or relationship.
Implement	Implement strategies and operations to solve problems involving familiar mathematical concepts and procedures.

Reasoning

Reasoning mathematically involves logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to problems set in novel or unfamiliar situations. Such problems may be purely mathematical or may have real-life settings. Both types of items involve transferring knowledge and skills to new situations; and interactions among reasoning skills usually are a feature of such items.

Even though many of the cognitive skills listed in the reasoning domain may be drawn on when thinking about and solving novel or complex problems, each by itself represents a valuable outcome of mathematics education, with the potential to influence learners' thinking more generally. For example, reasoning involves the ability to observe and make conjectures. It also involves making logical deductions based on specific assumptions and rules, and justifying results.

Analyze	Determine, describe, or use relationships among numbers, expressions, quantities, and shapes.
Integrate/Synthesize	Link different elements of knowledge, related representations, and procedures to solve problems.
Evaluate	Evaluate alternative problem solving strategies and solutions.
Draw Conclusions	Make valid inferences on the basis of information and evidence.
Generalize	Make statements that represent relationships in more general and more widely applicable terms.
Justify	Provide mathematical arguments to support a strategy or solution.